Devices to Illustrate Lissajous Figures

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Devices to Illustrate Lissajous Figures

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Longtime readers of *The Physics Teacher* will have noted my interest in oscillations and waves.\(^1\) In particular, Lissajous figures, which are the resultant of two simple harmonic motions at right angles to each other, have fascinated me since I first saw a device drawing them at Amherst College in 1957. In this paper I want to show some of the apparatus that has been developed to display Lissajous figures. There is more than antiquarian interest here; some of these can be reproduced fairly easily today.

In 1857 Jules Antoine Lissajous (1833–1880) used apparatus very similar to that in Fig. 1 to demonstrate what came to be called Lissajous figures.\(^2\) The two tuning forks are arranged so that their planes of vibration are at right angles to each other. Attached to the ends of the tines of each fork are metal plates (for symmetry), one of which is faced with a mirror. Light from a lamp reflects from the mirror on the right, and the reflected light beam oscillates up and down. The beam is then reflected from the mirror in the adjacent fork that is vibrating from side to side, thus making the beam also oscillate in that plane. In the original demonstration the weak light from the oil lamp was observed through a small telescope, allowing only one person to see the Lissajous figures. Today we do the demonstration by reflecting the beam from a laser from the two mirrors onto a nearby wall. This apparatus, in the apparatus collection of Marietta College in Ohio, dates from about 1900.

The heavy masses clamped to the tines of the tuning forks in Fig. 1 serve two purposes. The frequency

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**Fig. 1.** The basic optical setup for producing Lissajous figures; the apparatus is at Marietta College.

**Fig. 2.** A simple way of mounting two tuning forks at right angles to each other. The apparatus is at St. Mary’s College in Indiana.
of the forks is decreased when the masses are at the ends of the tines, and increases as they move in. The system may then be adjusted to give frequency ratios of the two forks that are the ratio of small numbers, producing the familiar simple Lissajous figures. Second, extra energy is necessary to set the large masses into motion, and thus the system runs for a longer period before the motion damps out.

The simple apparatus in Fig. 2, made by Max Kohl of Chemnitz, Germany, and sold for 66 marks ($15) in 1900, suggests a useful method for mounting two tuning forks at right angles to each other. Here the mirrors are attached to the sides of the tuning forks. For a one-to-one frequency ratio, choose two forks that have the same frequency and vibrate for a reasonably long time before damping away. This apparatus is at St. Mary’s College in Notre Dame, Ind.

The same geometry would serve to hold two short, stiff hacksaw blades so that they vibrate at right angles to each other. Small mirrors mounted on the ends of the blades reflect the light from a laser pointer. A pair of 6-in flexible steel rulers could be used in place of the hacksaw blades.

The problem of damping is solved with the apparatus in Fig. 3. The tuning forks are electrically driven so that they vibrate at a constant amplitude. The mechanism used to drive the forks is exactly the same as was used in induction and Ford-type coils (familiar from their use in the PSSC course): A direct current passing through a make-and-break contact alternately energizes an electromagnet, thus pulling the steel fork inward, and then releases it. The tuning forks, with tines 23.5 cm long, have a basic frequency of 128 Hz, and this may be lowered with the aid of the sliding masses. At the back is a device once used to hold a light source, and a previous owner mounted a lens on the base of the apparatus. The light source system can be moved right and left with a crank. This apparatus is listed at 435 marks in the 1932 Max Kohl catalogue. At present it sits on the floor of my dining room, a menace to unwary toes.

In 1827 Charles Wheatstone (1802–1875) invented the kaleidophone, a wire with one end firmly clamped and the free end polished into a convex spherical mirror. When the wire is pulled sideways and released, the image in the mirror of a nearby bright light source traces out Lissajous figures. Wheatstone used the same idea in his “apparatus for mechanically compounding two rectangular vibratory movements” in Fig. 4. The quoted name is from the 1883 catalogue of Rudolph Koenig of Paris, where it is listed at 200 francs ($40). As the crank is turned, the (missing) belt drives the small pulley in the center. An eccentric drives the bottom of the upright rod back and forth parallel to the base. At the same time, a small wheel attached to the horizontal shaft running along the front of the base picks off the rotary motion, and a second eccentric drives the bottom of the up-
right rod in SHM parallel to the short side of the base. At the top of the rod, normally adjusted so that it is pivoted halfway up its length, is a shiny bead, used to form the inverted virtual image of a light source. The apparatus is at Harvard University and is the only one I have seen in my travels.

A classic harmonograph, dating back to 1871, is shown in Fig. 5. The two pendula are pivoted on knife edges and swing at right angles to each other with relatively small amplitudes. A stylus linked to one pendulum draws the Lissajous figure on a piece of paper fixed to a platform atop the other pendulum. So far, this is the classic machine designed by Tisley and described by Whitaker and Romer. But wait! There is more! The hand-driven capstan in the middle of the base has a cord wound around it that can steadily draw up the mass on the left-hand pendulum, thus decreasing its period with time. The platform on which the paper rests rotates at a steady rate, driven by a spring motor. Since circular motion is a Lissajous formed by two SHMs of the same frequency and amplitude but with a phase difference of 90°, we now have two SHMs in each of two perpendicular directions, with one of the motions having a time-varying amplitude. The results are complex.

This looks like a device that could be reproduced fairly easily, using inexpensive geared-down motors instead of the capstan and the spring motor. To give the scale, the base of the device was about 80 cm above floor level.

The apparatus in Fig. 6 recently came into my personal collection. The mirrors are held on two pairs of taut wires stretched at right angles to each other. Displacing a mirror and then releasing it results in an oscillatory motion with a frequency that depends on the spacing of the pair of wires, their length, their tension, and the moment of inertia of the mirror. The wires are in one piece, with one end carried around a round brass rod to keep the tension the same on both halves. The apparatus is quite small, measuring 18 cm across the front and about 18 cm high. By using a handheld pocket laser pointer, it produces quite useful Lissajous figures. It was made by Max Kohl sometime after 1920.

A version of the apparatus that can be easily reproduced is shown in Fig. 7, taken from the 1890 edition of Hopkins’ Experimental Science. Instead of parallel wires, the mirrors are mounted on two parallel rubber bands. The small mirrors have metal clips glued
(epoxied) to their backs to allow them to be attached to the rubber bands that are held by screw hooks in the wooden framework. The cross-rods attached to the back of the mirrors increase the moment of inertia of the vibrating system and hence decrease the frequency. The optical system shown in the illustration is not needed when a laser pointer is used as the light source.

For a number of years I have used the modern commercial device in Fig. 8 to demonstrate Lissajous figures to large numbers of students. The device contains two stepper motors mounted with their shafts at right angles to each other. Internal electronic oscillators whose frequencies can be adjusted drive the motors, which have tiny mirrors glued to their shafts. I use a 0.5-mW He-Ne laser as a light source, primarily because it has a stable base. The two channels of a stereo audio signal may also be used to drive the stepper motors, producing Op Art effects.

For ease of construction, nothing surpasses the bent-arm kaleidophone. This device, shown in Fig. 9, is nothing more than a bent coat-hanger wire pushed into a base made from a length of two-by-four. A small mirror is glued onto a short piece of dowel pushed onto the free end of the wire. The transverse simple harmonic motion is due to twisting of the upper portion of the wire, and the vertical motion is due to the bending of this segment. A laser beam reflecting from the mirror traces out the Lissajous figures on a nearby wall.

References


2. The idea of combining two SHMs at right angles to each other is due to Nathaniel Bowditch (1773–1838) in a paper published in 1815.


8. The Laser Pattern Generator, sold by Arbor Scientific, P.O. Box 2750, Ann Arbor, MI 48106-2750; http://www.arborsci.com.

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