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Nuh Aydin
Kenyon College, aydinn@kenyon.edu

Erdogan Halat
Afyon Kocatepe University, ehalat@aku.edu.tr

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THE IMPACTS OF UNDERGRADUATE MATHEMATICS COURSES ON COLLEGE STUDENTS' GEOMETRIC REASONING STAGES

Nuh Aydin[1]
Kenyon College, Ohio
&
Erdogan Halat[2]
Afyon Kocatepe University, Turkey

ABSTRACT

The purpose of this study is to investigate possible effects of different college level mathematics courses on college students' van Hiele levels of geometric understanding. Particularly, since logical reasoning is an important aspect of geometric understanding, it would be interesting to see whether there are differences in van Hiele levels of students who have taken non-geometry courses that emphasize or focus on logic and proofs (Category I) and those that don't (Category II). We compared geometric reasoning stages of students from the two categories. One hundred and forty nine college students taking various courses from the two categories have been involved in this study. The Van Hiele Geometry Test designed to find out students' van Hiele levels was used to collect data. After the collection and analysis of the quantitative data, the participants' van Hiele levels are reported and the reasoning stages of two groups are compared. The results show that students taking logic/proof based courses attain higher reasoning stages than students taking other college level mathematics courses, such as calculus. The results may have implications that are of particular interest to teacher education programs. Finally, the results also confirm a previous assertion about correlation between van Hiele levels and proof writing.

Key Words: van Hiele levels; logic; mathematics courses; college students; geometry; teacher education programs

[1] Department of Mathematics
Kenyon College
Gambier, OH 43022
E-mail: aydinn@kenyon.edu
Phone: 740-427-5674
Fax: 740-427-5573

[2] Afyon Kocatepe University
College of Education
Department of Middle & Elementary Education
A.N.S Campus
AFYON / TURKEY
E-mail: ehalat@aku.edu.tr

INTRODUCTION

Since the mid 1980s there has been a growing interest in the area of teaching and learning geometry (e.g., Crowley, 1987; Gutierrez, Jaime, & Fortuny, 1991; Clements & Battista, 1990; Mason, 1997; Lappan, Fey, Fitzgerald, Friel, & Phillips, 1996; Halat, 2006/2007). The National Council of Teacher of Mathematics (NCTM) (2000) recommends that new ideas, strategies, and research findings be utilized in teaching in order to help students overcome their difficulties in learning mathematics. Knowledge of theoretical principles gives teachers an opportunity to devise practices that have a greater possibility of succeeding (e.g., Swafford, Jones, & Thornton, 1997). Based on over twenty years of research, the van Hiele theory is a well-known and well-regarded theory in geometry, structured and developed by Pierre van Hiele and Dina van Hiele-Geldof between 1957 and 1986. It has its own reasoning stages and instructional phases in geometry.

The van Hieles described five levels of reasoning in geometry. These levels, hierarchical and continuous, are level-I (Visualization), level-II (Analysis), level-III (Ordering), level-IV (Deduction), and level-V (Rigor) (Van Hiele, 1986).

Description of the levels:

Level-I: Visualization or Recognition. At this level students recognize and identify geometric figures according to their appearance, but they do not understand the properties or rules of figures. For example, they can identify a rectangle, and they can recognize it very easily because of its shape, which looks like the shape of a window or the shape of a door.

Level-II: Analysis. At this level students analyze figures in terms of their components and relationships among components and perceive properties or rules of a class of shapes empirically, but properties or rules are perceived as isolated and unrelated. A student should recognize and name properties of geometric figures.

Level-III: Ordering. At this level students logically order and interrelate previously discovered properties and rules by giving informal arguments. Logical implications and class inclusions are understood and recognized.

Level-IV: Deduction. At this level students analyze relationships of systems between figures. They can prove theorems deductively, construct proofs, and they can understand the role of axioms and definitions. A student should be able to supply reasons for steps in a proof.

Level-V: Rigor. At this level students are able to analyze various deductive systems like establishing theorems in different axiomatic systems, and they can compare these systems. A student should be able to know, understand and give information about any kind of geometric figures (e.g., Crowley, 1987; Fuys, Geddes, and Tischler, 1988).

The existence of level-0 is the subject of some controversy (e.g., Usiskin, 1982; Burger & Shaughnessy, 1986). Van Hiele (1986) does not talk about or acknowledge the existence of such a level. However, Clements and Battista (1990) have described and defined level-0 (Pre-recognition) as "Children initially perceive geometric shapes, but attend to only a subset of a shape's visual characteristic. They are unable to identify many common shapes" (p. 354). For example, learners may see the difference between triangles and quadrilaterals by focusing on the number of sides the polygons have but not be able to distinguish among any of the quadrilaterals (Mason, 1997).

EMPIRICAL RESEARCH ON THE VAN HIELE THEORY

Research has been completed on various components of this teaching and learning model. For instance, Wirszup (1976) reported the first study of the van Hiele theory, which attracted educators' attention at that time in the United States. In 1981, Hoffer worked on the description of the levels. Usiskin (1982) affirmed the validity of the existence of the first four levels in geometry at the high school level. In 1986, Burger and Shaughnessy focused on the characteristics of the van Hiele levels of development in geometry. They stated "students in the study who appeared to reason at different levels used different language and different problem solving processes on the tasks"(p.46). Furthermore, they said that students showed different levels of reasoning on different tasks. Fuys, Geddes, and Tischler (1988) examined the effects of instruction on a student's predominant Van Hiele level. Senk (1989), Mason (1997), and Gutierrez & Jaime (1998) evaluated and assessed the geometric abilities of students as a function of van Hiele levels. The study of Gutierrez, Jaime, & Fortuny (1991) with 9 eighth-grade pupils and 41 future primary school teachers was on an alternative way of analyzing the van Hiele levels of geometric thinking in the solid geometry. According to their study, most future primary teachers' van Hiele levels were level-I (recognition) and –II (analysis), but none of the participants showed level-IV (deduction) reasoning stage.

Mayberry (1983) conducted a study with 19 pre-service elementary school teachers. The tasks employed in her study were designed for the first four levels including seven geometric concepts that were squares, right triangles, isosceles triangles, circles, parallel lines, similarity, and congruence. According to the results of her study (1983), "the finding that 70% of the response patterns of the students who had taken high school geometry were below level-IV (deduction)" (p.68-69). In addition, the response of patterns showed that students who took part in the study were not at the suitable level to understand formal geometry, and that the instruction they had taken had not brought them to level IV (Deduction). The students' responses implied that the typical student in the study was not ready for a formal deductive geometry course (Mayberry, 1983).

Moreover, there have been some studies with pre-service elementary and secondary mathematics teachers regarding their reasoning stages in geometry. For instance, Knight (2006) conducted a research exercise with a total of 68 pre-service mathematics teachers, 46 elementary and 22 secondary. She found that the pre-service elementary and secondary mathematics teachers' reasoning stages were below level-III (informal deduction) and level-IV (deduction), respectively (Knight, 2006). Her findings are surprising because the van Hiele levels of pre-service elementary and secondary mathematics teachers are lower than the level expected of students completing grade 8 and grade 12, respectively. These results are consistent with the findings of Gutierrez, Jaime, & Fortuny (1991), Mayberry (1983), Duatepe (2000), and Olkun, Toluk, & Durmuş (2002). In all of these studies, none of the pre-service elementary and secondary mathematics teachers showed a level-V (Rigor) reasoning stage in geometry. Clearly, this is not a desirable outcome in teacher education.

According to van Hiele (1986), level-III is a transitional stage between informal and formal geometry. Geometry knowledge at this level is constructed by short chains of reasoning about properties of a figure and class inclusions. A student who functions at this level is able to follow a short proof based on properties gained from concrete experiences, but s/he is unable to construct a proof by her/himself. If students perform at the level-IV or -V geometry knowledge

then they will be able to do and write formal proofs. The study showed that although there is no individual van Hiele level that guarantees future success in proof writing, Level-III seems to be the critical entry level. Senk (1989) concluded, "the predictive validity of the van Hiele model was supported. However, the hypothesis that only students at level-IV or-V can write proofs was not supported" (p.309). According to Usiskin & Senk (1990) statements based on the study of Senk (1989), there was a positive correlation between students' van Hiele levels and proof writing success.

Usiskin & Senk (1990) expressed their surprise at the results of the Senk (1989)'s study about the positive correlation between van Hiele levels and proof writing. They said that the van Hiele geometry test (25-item multiple-choice test) could be used to predict the student's ability to write proofs. Van Hiele (1986) expressed two implications of the theory: a) students cannot show adequate performances at a level without having had experiences that enable students to reason intuitively at each preceding level. b) a student will not understand the instruction if the student's reasoning level is lower than the language of instruction. Mayberry (1983), Burger & Shaughnessy (1986) and Fuys et al. (1988) support statements (a) and (b). Van Hiele levels are hierarchical, and the progress from one level to the next is continuous. Furthermore, students' performance may vary from one concept to another in van Hiele theory. Concept formation in geometry may occur over long periods of time and requires specific interaction (Mayberry, 1983; Gutierrez et al., 1998). Moreover, Burger & Shaughnessy (1986) said that the van Hiele levels of reasoning could function as a basis for constructivist teaching experiments in geometry.

It is also shown that reform–based or NSF-funded standards-based curricula (e.g., Connected Mathematics Project, MATH Thematics, University of Chicago School Mathematics Project, Core-Plus Mathematics Project, and Everyday Mathematics) have more positive effects on students' learning of mathematics than conventional ones (cf., Fuson, Carroll, & Drueck, 2000; Romberg & Shafer, 2003; Reys, Reys, Lapan, Holliday, & Wasman, 2003; Senk & Thompson, 2003). Moreover, according to the Halat (2007), reform–based geometry curricula had a very favorable impact on the acquisition of the van Hiele levels and motivation in learning geometry.

Burger & Shaughnessy (1986) and Halat (2006) found mostly level-I reasoning in grades K-8 while Fuys et al. (1988) found no one performing above level-II in interviewing sixth and ninth grade average and "above average" students, which supports the idea that most younger students and many adults in the United States reason at levels-I (Visualization), –II (Analysis), -III (Ordering) and –IV (Deduction) of Van Hiele theory (e.g., Usiskin, 1982; Hoffer, 1986; Mayberry, 1983; Knight, 2006). Mayberry (1983) and Fuys, Geddes, & Tischler (1988) stated that content knowledge in geometry among pre-service and in-service middle school teachers is not adequate. There are many factors, such as gender, peer support, age, type of mathematics course, instruction, and so forth that appear to be affecting pre-service mathematics teachers' or college students' performance and motivation in mathematics.

The purpose of the Study

The aim of this current study was to investigate possible effects of different college level mathematics courses on college students' van Hiele levels. Particularly, since logical reasoning is an important aspect of geometric understanding, we were interested in testing whether there are differences in van Hiele levels of students who have taken non-geometry courses that emphasize or focus on logic and proofs (Category I courses) and those that don't (Category II courses). More information about the two categories is given in the method section.

Furthermore, Usiskin (1982), Mayberry (1983), Burger & Shaughnessy (1986) and Fuys et al. (1988) confirmed the validity of first four levels of the geometric thought (visualization, analysis, abstract, and deduction). They all agreed that the last level, rigor (level-V), was not often seen in high school students. It was more appropriate for college students. This study also aimed to examine this argument. Finally, the results of the study can be interpreted as supporting the previous assertion by Usiskin & Senk (1990) about correlation between van Hiele levels and proof writing.

METHOD

Participants

In this study the researcher followed the "convenience" sampling procedure defined by McMillan (2000), where a group of participants is selected because of availability. Participants in the study were 149 college students divided into two groups, group-I and -II. The group-I consisted of 41 students from Category I courses, those courses that directly use or teach logic or proof writing. Category I courses in this study consist of

a) An Introduction to Computer Programming course. This is an introductory course with no formal prerequisites. It teaches and uses the programming language C++.

b) An Introductory Course on Logic, Set Theory and Proof Techniques. This is a sophomore level course and is required for mathematics majors and minors.

c) A number of advanced mathematics courses that have the introductory proof writing course (Category I-b course above) as a pre-requisite, such as Real Analysis, Abstract Algebra, and other advanced elective mathematics courses (including a course on Euclidean and non-Euclidean geometry which had only 5 students).

The group-II included 108 students taking Category-II mathematics courses, those courses that do not directly use or teach logic or proof writing. In this study, Category II courses included the following courses

a)   An introductory course on statistics

b)   Each one of the three semesters of calculus

It is important to note that

- almost all participating students in both categories took geometry in high school
- None of the students took a geometry course at college level (with a small exception described below)
- None of the courses involved in the study directly teaches any geometry content, except for an advanced mathematics course on Euclidean and non-Euclidean geometry which only had 5 students. This course is only offered every two or three years at this college.

The study took place in a small liberal arts college in a Midwestern state of the US.

The general student profiles for each group were as follows:

Category I-a: There were 12 students in this group. About 40% of students were females, and about 60% were males. Most of the students were first year students, though there were a few students from each class. There were no students who declared mathematics as a major in this course. Several of them were undecided and the rest of them had various non-mathematics majors from a variety of divisions of the liberal arts disciplines.

Category I-b: This was a small group of 6 students with the male-female ratio being exactly equal. All students were either mathematics majors or minors (or strongly considering one).

Category I-c: This group consisted of 23 students. Most of the students were juniors or seniors, with few sophomores. Virtually all of them were mathematics majors (in many cases they were double majors with one of the sciences, Economics, or English). The male-female ratio was around 65% to 35% favoring males.

Category II-a: Though some mathematics majors do take this introductory course, they were not included in this group of size 47. So none of the students included in this group had mathematics as a major, neither did they take any advanced mathematics courses (or any courses from category I). The male-female ratio was around 66% to 34% favoring males. Majority of students were first or second year students, with a few students from the upper classes. There were some undecided students and declared majors spanned a wide spectrum of disciplines.

Category II-b: A total of 61 students included in this group. The male-female ratio was almost equal and a great majority of students were first or second year students. A small percent of students, most of them from the third semester multivariable Calculus, were declared mathematics majors.

Data Sources

The researchers gave participants a geometry test called *Van Hiele Geometry Test (VHGT)*. The VHGT was administered to the participants by the researchers during a single class period. The *Van Hiele Geometry Test (VHGT)* consists of 25 multiple-choice geometry questions. The VHGT is designed to measure students' van Hiele levels in geometry (Usiskin, 1982). The VHGT was given to the participants at or near the completion of the courses at the end of the semester Fall-2006. Due to time limitations and other constraints, we were not able to administer pre-tests to the participants. This is a limiting factor on the conclusions and implications of the study. While we get interesting suggestions from the results of the study, the reader should be cautious about making generalizations from the results in this study. Nevertheless, this study poses some questions and issues for further investigation. A few possible ways to strengthen the study are discussed at the end.

Test Scoring Guide

In this study, the 1-5 scheme was used for the levels. This scheme allows the researchers to use level-0 for students who do not function at what the van Hieles named the ground or basic level. It is also consistent with Pierre van Hiele's numbering of the levels. For this report, all references and all results from research studies using the 0-4 scale have been changed to the 1-5 scheme.

All participants' answer sheets from VHGT were read and scored by the investigators. All participants received a score referring to a van Hiele level from the VHGT guided by Usiskin's grading system.

"For Van Hiele Geometry Test, a student was given or assigned a weighted sum score in the following manner:

- 1 point for meeting criterion on items 1-5 (level-I)
- 2 points for meeting criterion on items 6-10 (level-II)
- 4 points for meeting criterion on items 11-15 (level-III)
- 8 points for meeting criterion on items 16-20 (level-IV)
- 16 points for meeting criterion on items 21-25 (level-V)" (1982, p. 22)

Analysis of Data

        The data were responses from students' answer sheets. In the process of the assessment of participants' van Hiele levels, the criterion for success at any given level was four out of five correct responses. The researchers ran the independents-samples t-test to compare two groups' van Hiele levels and to see the effects of the courses from both categories on the participants. Then they constructed frequency tables to get detailed information about distributions of participants' van Hiele levels.

RESULTS

        Table 1 presents the descriptive statistics and the independent samples t-test for college students' van Hiele levels in both groups, Category-I and –II. According to the table 1, the mean score of group-I (4.02) is numerically higher than that of group-II (2.64). The independent-samples t-test showed that the difference between the groups is statistically significant, [$p < .001$, significant at the $\alpha/2 = .025$ using critical value of $t_{\alpha/2} = 1.96$], favoring the students who took Category-I mathematics courses.

Table 1

Descriptive Statistics and the Independent-Samples T-Test for Students' van Hiele Levels

	N	M	SD	SE	df	t
Category-I	41	4.02	.90	.14	99.010	7.45*
Category-II	108	2.64	1.24	.12		
Total	149					

Note. *$p < .001$, significant at the $\alpha/2 = .025$ using critical value of $t\alpha/2 = 1.96$.

        According to Burger & Shaughnessy (1986), the progress through the levels is continuous and not discrete. Despite the fact that students generally are assigned to a single van Hiele level, there may be students who cannot be assigned to a single van Hiele level. Gutierrez, Jaime, & Fortuny (1991) used a 100 - point numerical scale to determine the van Hiele levels of students who reason between two levels. This numerical scale is divided into five qualitative scales: "'Values in interval' (0%, 15%) means 'No Acquisition' of the level. 'Values in the interval' (15%, 40%) means 'Low Acquisition' of the level. 'Values in the interval' (40%, 60%) means 'Intermediate Acquisition' of the level. 'Values in the interval' (60%, 85%) means 'High Acquisition' of the level. Finally, 'values in the interval' (85%, 100%) means 'Complete Acquisition' of the level'" (p. 43).

        The mean score 4.02 of the group-I can be explained with the scale described above. The score .02 can be placed into the interval named "No Acquisition" of the upper level. In other words, students who were in the group-I completed the level-IV (Deduction), but they have not attained the level-V (Rigor). At level-IV, students analyze relationships of systems between figures. They can prove theorems deductively, construct proofs, and they can understand the role of axioms and definitions. A student should be able to supply reasons for steps in a proof.

On the other hand, the interpretation of the mean' score 2.64 for the group-II would be that students' average van Hiele level falls between levels-II (Analysis) and–III (Informal Deduction) . Using the interval scale, the .64 indicates that there is high acquisition of level -III understanding, but not completed.

Table 2 indicates the participants' reasoning stages in detail. According to the frequency table 2 below, none of the students in group-I showed levels-0 (pre-recognition) and –I (visualization) reasoning stages. Mostly they demonstrated higher levels of thinking, level-IV (34.1%) and –V (36.6%) (see figure 1 below). However, students in group-II showed all geometric thinking stages in different percentiles. Mostly they showed level-III (47.2%) geometry knowledge on the test (see Figure 2).

Table 2
Frequency Table for Students' van Hiele Levels

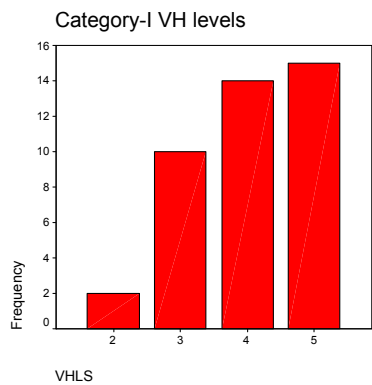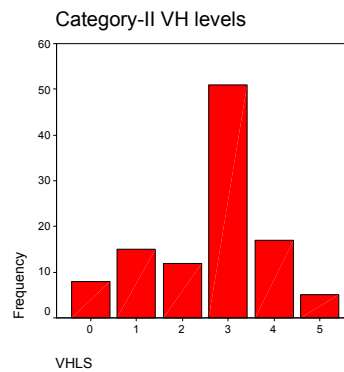| Groups	N	Level-0	Level-I	Level-II	Level-III	Level-IV	Level-V
		%	%	%	%	%	%
Category-I	41	0	0	4.9	24.4	34.1	36.6
Category-II	108	7.4	13.9	11.2	47.2	15.7	4.6
Total	149						

Figure 1

Figure 2

DISCUSSION & CONCLUSION

Students' Van Hiele Levels

Usiskin (1982), Mayberry (1983), Burger & Shaughnessy (1986) and Fuys et al. (1988) confirmed the validity of first four levels of the geometric thought (visualization, analysis, abstract, and deduction). Burger & Shaughnessy (1986) and author (2007) found mostly level-I reasoning in grades K-8 while Fuys et al. (1988) found no one performing above level-II in interviewing sixth and ninth grade average and "above average" students, which supports the idea that most younger students and many adults in the United States reason at levels-I (Visualization), –II (Analysis), -III (Ordering) and –IV (Deduction) of Van Hiele theory (i.e., Usiskin, 1982; Hoffer, 1986; Knight, 2006). They all agreed that the last level, rigor (level-V),

was not suitable for high school students. They stated that it was more appropriate for college students.

However, there were some studies with college students, or pre-service elementary and secondary mathematics teachers regarding their reasoning stages in geometry (e.g., Gutierrez, Jaime, & Fortuny, 1991; Mayberry, 1983; Duatepe, 2000; Olkun, Toluk, & Durmuş, 2002; Knight, 2006). These studies showed that none of the pre-service elementary and secondary mathematics teachers showed a level-V (Rigor) reasoning stage in geometry. This result is in contrast with the argument mentioned above. According to this current study, on the other hand, there were some college students (36.6%) in group-I and (4.6%) in group-II showing level-V (Rigor) reasoning stages.

Based on over twenty years of research, the van Hiele theory is a well-structured and well-known theory having its own reasoning stages and instructional phases in geometry. Many researchers have studied and confirmed different aspects of the theory since proposed by the van Hieles. This current study adds to the set of studies by examining the validity of the level-V (Rigor).

This study supports the research findings claiming that level-V (Rigor) is more appropriate for college students than for high school students. The results can also be interpreted as confirming the previous assertion about correlation between van Hiele levels and proof writing abilities of students.

The Impacts of Taking Higher Level Mathematics Courses on College Students' Van Hiele Reasoning Stages

The main point of this current study was to examine possible effects of different college level mathematics courses on college students' van Hiele levels. Particularly, since logical reasoning is an important aspect of geometric understanding, we were interested in testing whether there are differences in van Hiele levels of students who have taken non-geometry courses that emphasize or focus on logic and proofs (Category I courses) and those that don't (Category II courses).

The analysis of the data revealed that students taking logic/proof based courses attain higher reasoning stages than students taking other college level mathematics courses. The difference between two groups might be attributable to such factors as students' pre-existing knowledge, age, types of courses, and so forth. None of the participants have taken a geometry course since high school, except for five students. Moreover, we collected data for participants' SAT/ACT Math scores in the introductory courses (for the advanced mathematics courses we decided that information was not necessary). The results suggest that SAT-Math scores cannot fully explain the difference. For example, comparing the introductory programming course and second semester of Calculus, the average SAT-Math score in the programming course (704) is lower than the average SAT-Math score in Calculus II (~710), yet average van Hiele level in the programming course is higher. A similar comparison exists between average GPAs of participants in the two courses. We consider this to be an indication that logic/proof based courses enhance students reasoning stages.

Being in different levels in terms of ages or years in school might influence students reasoning stages. For example, most of the students taking category-I courses were juniors or seniors, except for the students in the introductory programming course, but students taking category-II courses were mostly first or second year students. However, when we look at the introductory course from Category I, that is the computer programming course, the ages/years of

the students in that course is comparable to those in Calculus I yet the van Hiele levels of students in the programming course is significantly higher than that of students in Calculus I, 4.09 and 2.62 respectively. On the other hand, according to Fuys, Geddes, & Tischler (1988), students' success in mathematics depends on instruction more than student's age or biological maturation. Putting these two findings together, the difference in terms of geometric reasoning stages between the two groups may be more attributable to the impact of the courses than the ages of the students.

Implications for Teacher Education Programs

This current research has several possible suggestions for both instructors and pre-service teacher education programs. According to Usiskin & Senk (1990), there is a positive correlation between van Hiele levels and proof- writing success in geometry. The results of the current study support their conclusion, assuming that students in Category I courses have better proof writing abilities than students in Category II courses. Though we have not conducted formal measurement of proof writing abilities in either category of students, it is reasonable to assume that students in advanced mathematics courses, all of which require the logic/proof writing course as a prerequisite, who make up the majority of the students in Category I have much better facility in writing mathematical proofs. Moreover, although the Category-I courses are non-geometry courses and the contents of the courses in Category-I are not related to the Euclidean-geometry (except for the small class of 5 students in the geometry course), constructing formal proofs or dealing with logical issues greatly affected students geometric reasoning levels. Therefore, this study suggests that knowledge of students' van Hiele levels might help instructors to better anticipate their students' proof writing abilities.

Furthermore, several studies have shown that many of the prospective teachers do not attain a level of geometry that they are expected to teach (i.e., Gutierrez, Jaime, & Fortuny, 1991; Knight, 2006). This is clearly unacceptable. This study indicates that logic/proof based courses might have a strong positive impact on geometric understanding of students, even without any additional geometry content. Therefore, teacher education programs may want to consider adding such a course to their program requirements.

LIMITATIONS & FUTURE RECOMMENDATIONS

The findings of this study have a limited scope and should not be immediately generalized because of several reasons. Firstly, this study has been carried out in a small, private and selective liberal arts college in the US. Therefore, it would be useful if a future study tests whether similar conclusions hold at a larger scale.

Secondly, we have been able to administer only post-tests to the students. A way to strengthen this study would be to apply both a pre-test and a post-test to see the effects of individual courses more clearly. Regarding prospective teachers in education, a future study might want to specifically target students in such a program. It would be interesting to see if there is a pre-service mathematics teaching program where some students take logic/proof based courses and some not; and if there is such a program whether there is a difference between van Hiele levels of students in each group.

Thirdly, we note that most of the students in Category I courses are mathematics majors and most of the students in Category II are not (either they have different majors or are

undeclared). It would be interesting to investigate if similar results hold amongst students who are not mathematics majors or minors. Researchers might want to compare students taking logic courses , e.g. in philosophy, who have not taken college level mathematics courses with those who have taken neither logic nor mathematics courses in terms of their van Hiele levels.

Finally, the results may be interpreted as confirming a previous assertion by Usiskin & Senk (1990) about positive correlation between van Hiele levels and proof writing abilities of students. We consider the assumption of the hypothesis "students in advanced mathematics courses have better proof writing abilities than students in introductory mathematics courses" to be a reasonable one to take for granted. However, researchers of a future study may want to more explicitly measure the proof writing abilities of students in two categories and the correlation stated above.

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